Oxford Cambridge and RSA

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Question Set 2

1 In this question the horizontal unit vectors **i** and **j** are in the directions east and north respectively.

A model ship of mass 2 kg is moving so that its acceleration vector $\mathbf{a}$ m s^{-2} at time t seconds is given by $\mathbf{a} = 3(2t-5)\mathbf{i} + 4\mathbf{j}$. When $t = T$, the magnitude of the horizontal force acting on the ship is 10 N.

Find the possible values of T. **[4]**

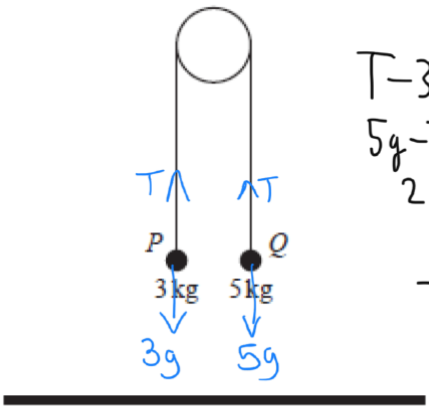
$10 = 2\begin{pmatrix} 6t-15 \\ 4 \end{pmatrix}$ $(12t-30)^2 = 36$

$12t-30 = \pm 6$

$t = 3, 2$

$10 = \begin{pmatrix} 12t-30 \\ 8 \end{pmatrix}$ $(12t-30)^2 + 8^2 = 10^2$

2 Particles P and Q, of masses 3 kg and 5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and P and Q are above a horizontal plane (see diagram).



$T - 3g = 3a$

$5g - T = 5a$

$2g = 8a$

$a = \frac{g}{4}$

$T = \frac{15g}{4} = 36.8\text{ N}$

(a) Find the tension in the string immediately after the particles are released. **[4]**

After descending 2.5 m, Q strikes the plane and is immediately brought to rest. It is given that P does not reach the pulley in the subsequent motion.

(b) Find the distance travelled by P between the instant when Q strikes the plane and the instant when the string becomes taut again. **[4]**

$S = 2.5$ $\quad$ $S = ?$

$U = 0$ $\quad$ $U = \frac{\sqrt{5g}}{2}$

$V = ?$ $\quad$ $V = 0$

$A = \frac{g}{4}$ $\quad$ $A = -g$

$T =$ $\quad$ $T =$

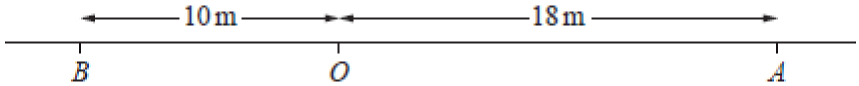
$V^2 = U^2 + 2as \rightarrow$

$V^2 = \frac{5}{4}g$ $\quad$ $0 = \frac{5g}{4} - 2gs$

$V = \frac{\sqrt{5g}}{2}$

$S = \frac{5}{8}\text{ m}$

3



A particle P is moving along a straight line with constant acceleration. Initially the particle is at O. After 9 s, P is at a point A, where $OA = 18$ m (see diagram) and the velocity of P at A is $8\,\text{m s}^{-1}$ in the direction $\overrightarrow{OA}$.

(a) (i) Show that the initial speed of P is $4\,\text{m s}^{-1}$. [2]

$S=18$ $\quad s=\left(\frac{u+v}{2}\right)t$
$U=$
$V=8$ $\quad \frac{36}{9}=u+8$
$A=$ $\quad u=-4$ speed $=4$
$T=9$

(ii) Find the acceleration of P. [2]

$V=u+at$
$8=-4+9a$ $\quad a=\frac{4}{3}$

B is a point on the line such that $OB = 10$ m, as shown in the diagram.

(b) Show that P is never at point B. [4]

$S=-10$ $\quad -10=-4t+\frac{1}{2}\left(\frac{4}{3}\right)t^2$
$U=-4$ $\quad 2t^2-12t+30=0$
V
$A=1.33$ $\quad t^2-6t+15=0$ $\therefore$ no solutions
$T=?$

A second particle Q moves along the same straight line, but has variable acceleration. Initially Q is at O, and the displacement of Q from O at time t seconds is given by

$$x = at^3 + bt^2 + ct,$$

where a, b and c are constants.

$v=3at^2+2bt+c$
$a=6at+2b$

It is given that
- the velocity and acceleration of Q at the point O are the same as those of P at O,
- Q reaches the point A when $t = 6$.

$t=0,\ v=-4$
$-4=3a(0)^2+2b(0)-4$
$c=-4$
$t=0,\ a=\frac{4}{3}$
$\frac{4}{3}=6a(0)+2b$
$b=\frac{2}{3}$
$t=6,\ x=18$
$18=a(6)^3+\frac{2}{3}(6)^2+(-4)(6)$
$a=\frac{1}{12}$

$v=\frac{1}{4}t^2+\frac{4}{3}t-4$

$t=6$ $\quad v=\frac{1}{4}(6)^2+\frac{4}{3}(6)-4$

$=13\ \text{ms}^{-1}$

Total Marks for Question Set 2: 25